Published on http://www.matrixreasoning.com
Full list of publications: http://www.matrixreasoning.com/publications/

Chaos as a Subject of Exact Science.
1. Fractal geometry of Public Opinion

Leonid Andreev
Equicom, Inc., Scottsdale, AZ 85255, U.S.A., equicom@matrixreasoning.com
http://www.matrixreasoning.com

Emergence of chaotic state in systems of any degree of complexity can be detected by the proposed method involving monotonous changes in dimensions of a system's parameters. The extent of disorder in a system is assessed based on egoentropy, a criterion of complexity, previously proposed by us. It has been shown that any given system can be characterized by two limit egoentropy values: initial egoentropy (E_0) and egoentropy in the state of maximal chaos (E_{max}).

It is experimentally demonstrated that a system's transition to the state of chaos is manifested in a spike of chaos emerging at a certain point of monotonous changes in dimension values of a system's parameters. A typical chaotic spike has a complex structure: in the state close to maximal chaos, there is a response in the form of a narrow inverse peak, reflecting the emergence of order (antichaos) amid ultimate chaos. After reaching a certain level of order, the system regains the extent of chaos predetermined by the dynamics expressed in the ascending curve of the chaotic spike.

The proposed methodology is demonstrated by the example of analysis of a public opinion poll. It is shown that the emergence of chaotic spikes in response to the monotonous changing of dimensions of the system's parameters can be accurately computed, hence predicted.

Introduction

This paper is the first one in the "Chaos Study as Exact Science" series of publications on this web site. We will demonstrate that the emergence of chaos in systems can be accurately predicted, and that such predictions can be effectively utilized in various fields of research and practice. The examples chosen by us to demonstrate and explain the investigations underlying the proposed methodology are based on simple systems of objects – however, not due to following the tradition of classical science to start with simple objects and gradually advance into investigations on more complex levels. The choice of objects of these experiments has been based on an entirely different idea: to demonstrate that with a proper technology, potential chaos can be identified in practically any system, even in most simple ones, where, by common sense, the probability of emergence of a generalized chaotic reaction is low.

Previously, we proposed a new criterion of complexity – egoentropy – and demonstrated its efficiency in detecting chaotic spikes that emerge in complex systems upon induced monotonous changes in parameter dimensions [1]. In this work, we investigate the fine structure of chaotic spikes and demonstrate that their emergence can be predicted with high accuracy.

This investigation has been performed with the use of a data set previously referred to in our paper on cluster spectroscopy [2]: selected data from The Los Angeles Times National Poll (Study # 443, July 31, 2000, published same year on http://www.latimes.com and reproduced here with courteous permission of Ms. Susan Pinkus, Director of The Los Angeles Times Poll). In this paper, we do not offer a final interpretation of the poll results – the data have been used by us with the sole purpose of demonstration of the subject methodology and the new opportunities it provides to experts interested in new methods for fine analysis and interpretation of this type of information.

Table 1. Responses to question *"What is your personal view on gun control laws? Generally speaking, do you think they ought to be more strict than they are now, or less strict, or do you think the laws we have now are about right?"*, from The Los Angeles Times National Poll, Study # 443, July 31, 2000 (*"don't know"* answers not included).

	More strict	Less strict	About right
Republicans	31	11	54
Conservatives	36	15	45
Men	41	12	44
Independents	54	7	34
Moderates	59	3	35
Women	64	4	28
Democrats	69	4	24
Liberals	69	4	23

In the analysis of this system, we applied the previously described egoentropy criterion [1] as a measure of disorder. The analysis was performed by using *MeaningFinder 2.2* software commercially available from Equicom, Inc., a free trial version is downloadable on http://www.matrixreasoning.com (see *Appendix* to this paper).

Results

Parameter dimension values changing from 0 to 1

A 'parameter dimension value' refers to the weight that can be assigned to each individual parameter describing a set of objects under analysis, in the course of a similarity matrix construction. A similarity matrix is constructed by hybridization of monomer similarity matrices computed based on each individual parameter. Hybridized similarity coefficients are computed as geometric means of monomer similarity coefficients. To assign a certain weight to a parameter, a respective monomer similarity coefficient is raised to the power equal to the necessary weight. Thus assigned weights of parameters represent their dimension values (*DV*) [1, 2, 3].

By gradually changing dimension values of parameters, one at a time, within the interval of 0 to 1, one can analyze a system within the range between its two states: from the point when the parameter is completely absent from description of the object, to the point when it is fully considered along with all other parameters. The results of such analysis of the system of objects and parameters shown in Table 1 were as follows. The changes in the dimension value (*DV*) of the "More strict" parameter did not produce any effect, whereas the changes in the "Less strict" parameter *DV* within the range from 0.118 to 0.146 caused a steep increase of the system's egoentropy (*E*) from the initial value of 1.16 to 1.82. As for the "About right" parameter, its weight changes resulted in the emergence of a chaotic spike whose peak maximum corresponded to *DV* of about 0.434, causing the egoentropy increase by 220 times as compared to the zero line (Fig. 1).

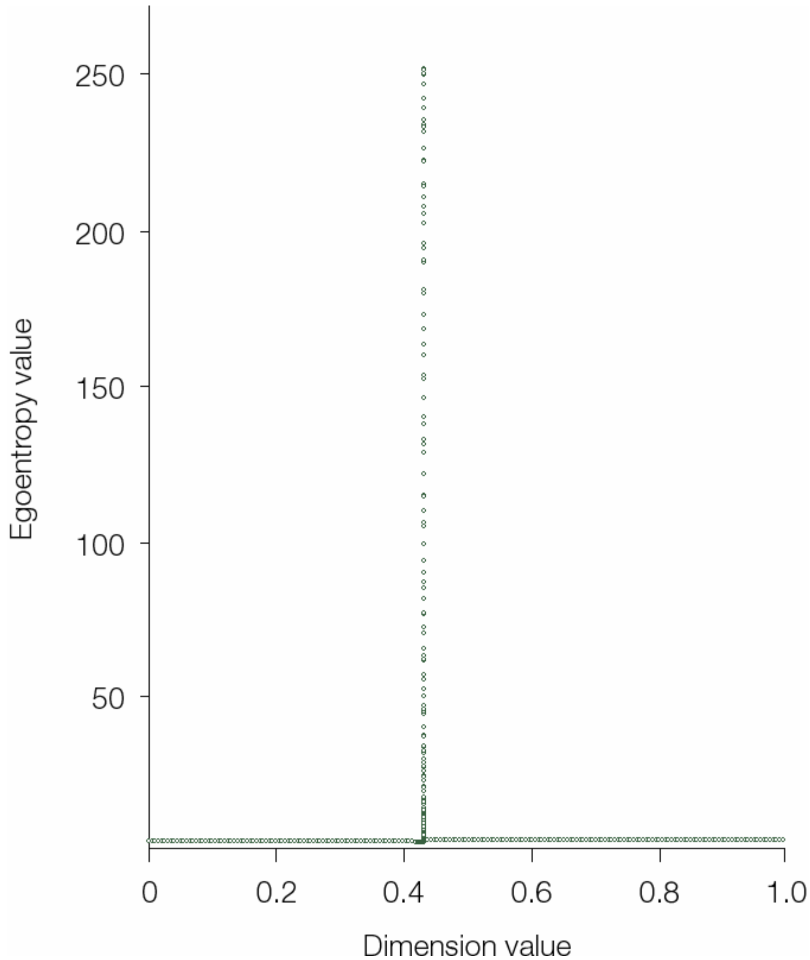

Figure 1. Chaotic spike emerging in the course of changing the dimension value of parameter "About right" from 0 to 1.

Analysis of fine structure of chaotic spikes. Antichaotic spikes

The spike shown in Fig. 1 is the manifestation of a sudden emergence of chaos in the system, in full conformity with accepted views on chaos. In forthcoming publications, we will discuss the causes that underlie the emergence of chaotic states in complex systems by examples of concrete analyses. In this paper, we focus mainly on the forms of chaotic spikes and on some of the issues of regularities in their emergence.

Fig. 2 demonstrates the fine structure of the chaotic spike shown in Fig. 1. To study it, we set the "About right" parameter *DV* to vary within a very narrow interval: from 0.433 to 0.435. As is seen in Fig. 2, the surge recorded in Fig. 1 consists, in fact, of two surges: at a point close to the maximum, the bell-shaped Gaussian curve abruptly collapses, thus forming a reverse spike whose extremum almost reaches the point that corresponds to the *E* value of a well-ordered system. The formation of this antichaotic spike occurs within an interval that equals $8 \cdot 10^{-5}$ of the parameter dimension, and its extremum point is located approximately at the center point of the interval.

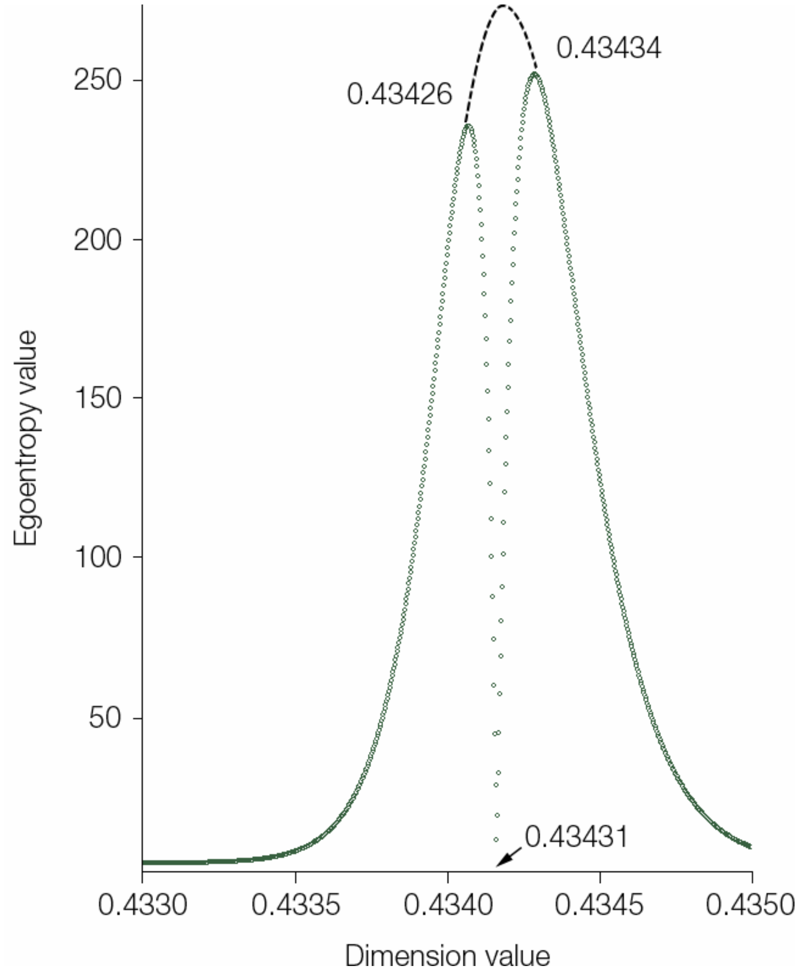

Figure 2. Same chaotic spike as in Fig. 1, registered at the *DV* scale 500:1. The black dotted part of the curve is the hypothetical trajectory of the chaotic spike in the absence of the antichaotic spike.

Our studies into the fine structure of chaotic spikes in various systems, both natural and artificial, demonstrated that the collapsing of chaotic spikes occurs in most diverse systems, and the underlying mechanism is as follows: changes in parameter dimensions can bring a system to maximum disorder possible in a given system under given conditions, and, if the dimension of an affected parameter continues to change, then at a certain point of such changes the elevated chaos starts transforming into order, i.e. the antichaotic state. In the same way as chaotic spikes emerge in highly organized environment, antichaotic spikes emerge amid high chaos.

Mechanism of emergence of chaotic response

Evolutionary transformation of similarity matrices [4] causes a dataset under processing to transform into two data points with similarity coordinates of 0 and 1, wherein "0" is a similarity between the points, and "1" is a similarity "within" each point. This is not the same as dataset bifurcation; instead, this phenomenon constitutes bifocal compression of data points [5]. Visualization of the process of data points' bifocal compression – showing how a community of hundreds of data points scattered across a diagram eventually get compressed, in the course of the evolutionary transformation of their similarity matrix, into two foci – is provided by the "Matrix Diagram" function of *MeaningFinder 2.2*.

However, in systems that are in the state of chaos, bifocal compression does not occur. For instance, in the example with the national poll on gun control laws, the system reaches a chaotic state when the "About right" parameter has *DV* of 0.434, in the result of which the object "Independents" appears to be equally similar to objects "Liberals", "Democrats", "Women", and "Moderates", as well as to "Republicans", "Conservative", and "Men", thus becoming the chaos factor. In cluster spectra, obtained by the method described in [2], such events are manifested in the form of abrupt rises of a zero line.

The cluster spectra shown in Figures 3A – 3C show three different states of the system under analysis. The spectrum in Fig. 3A corresponds to a normal process of bifocal

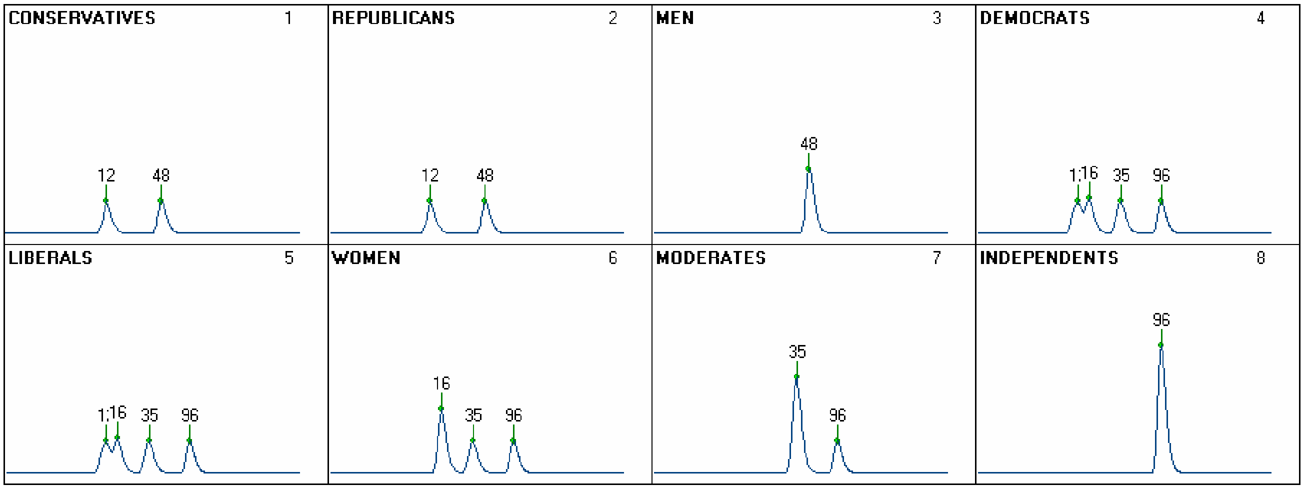

Figure 3A. Cluster spectra obtained based on the data in Table 1.

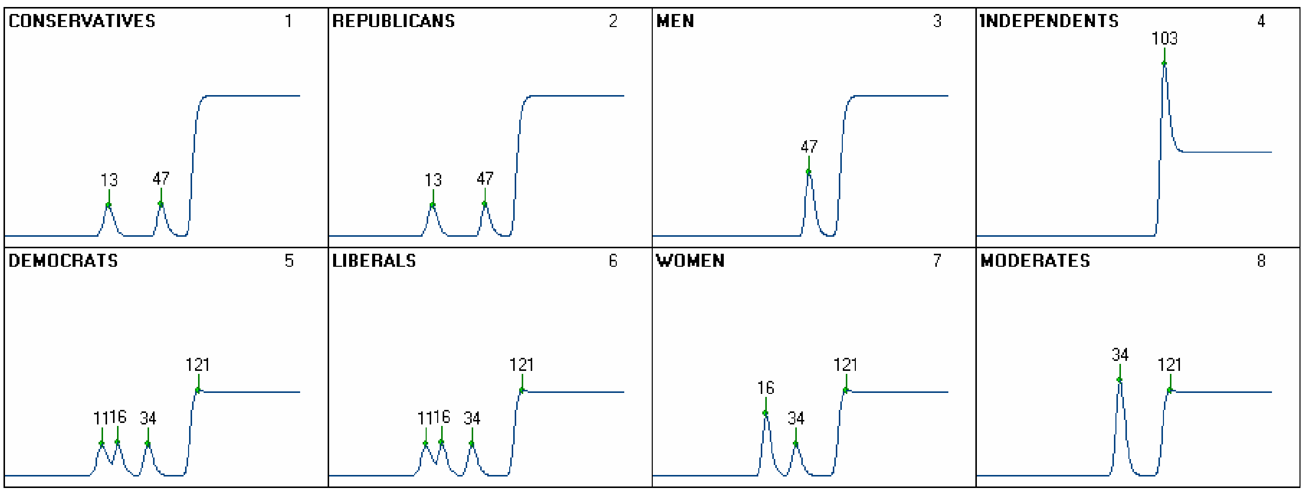

Figure 3B. Same as in Fig. 3A, but at *DV* of 0.434264 for parameter "About right".

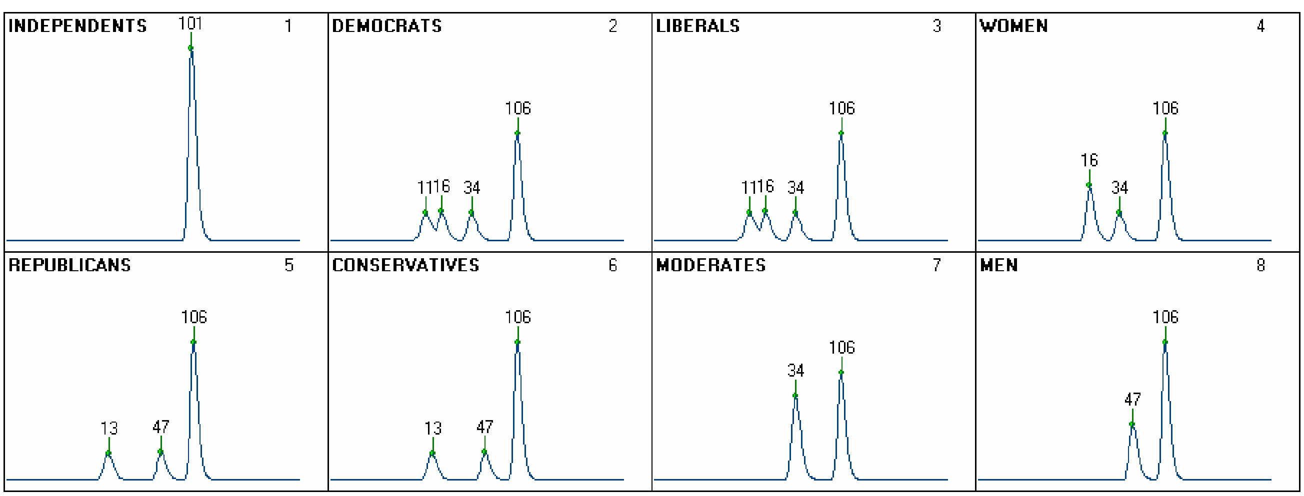

Figure 3C. Same as in Fig. 3A, but at *DV* of 0.43431346 for parameter "About right".

compression when all three parameters' dimension values are equal. The spectrum in Fig. 3B reflects the chaotic state of the system at *DV* = 0.434264. Fig. 3C shows the state of maximal antichaos that happens when *DV* of parameter "About right" equals 0.43431346, resulting in E = 5.9 which corresponds to a fairly well-ordered system (at any other dimension values for the same parameter, the system's egoentropy goes up). However, this system is only seemingly well-ordered. As one can see in Fig. 3C, one of the subclusters joins very different objects together (i.e. these objects do not resolve) – see the cluster spectra whose last peaks correspond to the 106th transformation; and another subcluster includes just one object – "Independents", and its formation is complete after 101 transformations. Thus, it is due to the object "Independents" that the antichaotic spike emerges amid chaos that prevents the resolution of all objects. Fig. 4 shows the combined cluster spectra taken in the state of chaos (DV = 0.434264, E = 247) and in the state of maximum antichaos, respectively.

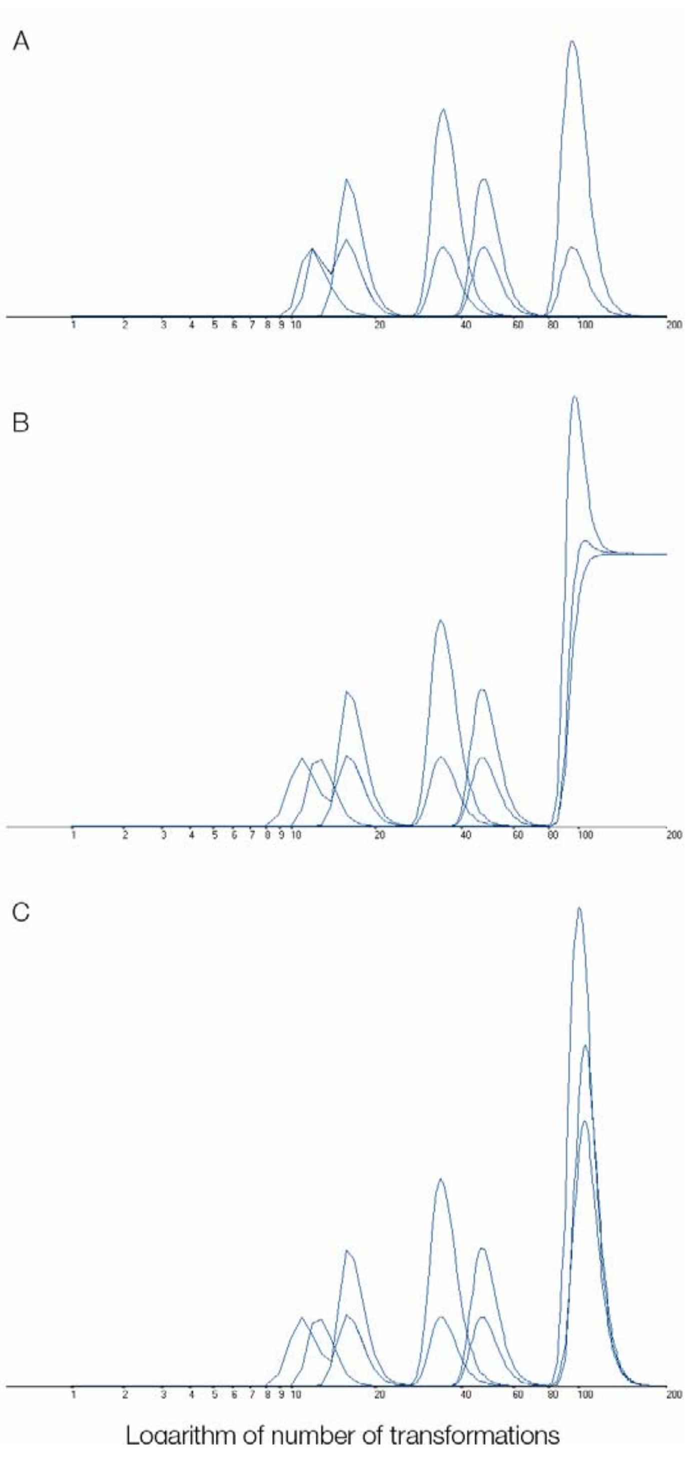

Figure 4. Combined cluster spectra obtained in conditions specified in legends to Fig. 3A, 3B and 3C, respectively.

Egoentropy dynamics upon DV changes above 1

Upon changes of each parameter's dimension value from 1 to 20, it appeared that the "More strict" parameter did not have any effect on the system's egoentropy. In case of parameter "About right", the *DV* variations within the range of 4.5 – 4.8 resulted in graduated descent of the zero line. Changes in the dimension value of parameter "Less strict" resulted in a sharp narrow spike at $DV = 1.744$, after which the zero line was constant throughout the following changes of the parameter *DV* up to 20. The above results are presented in Fig. 5.

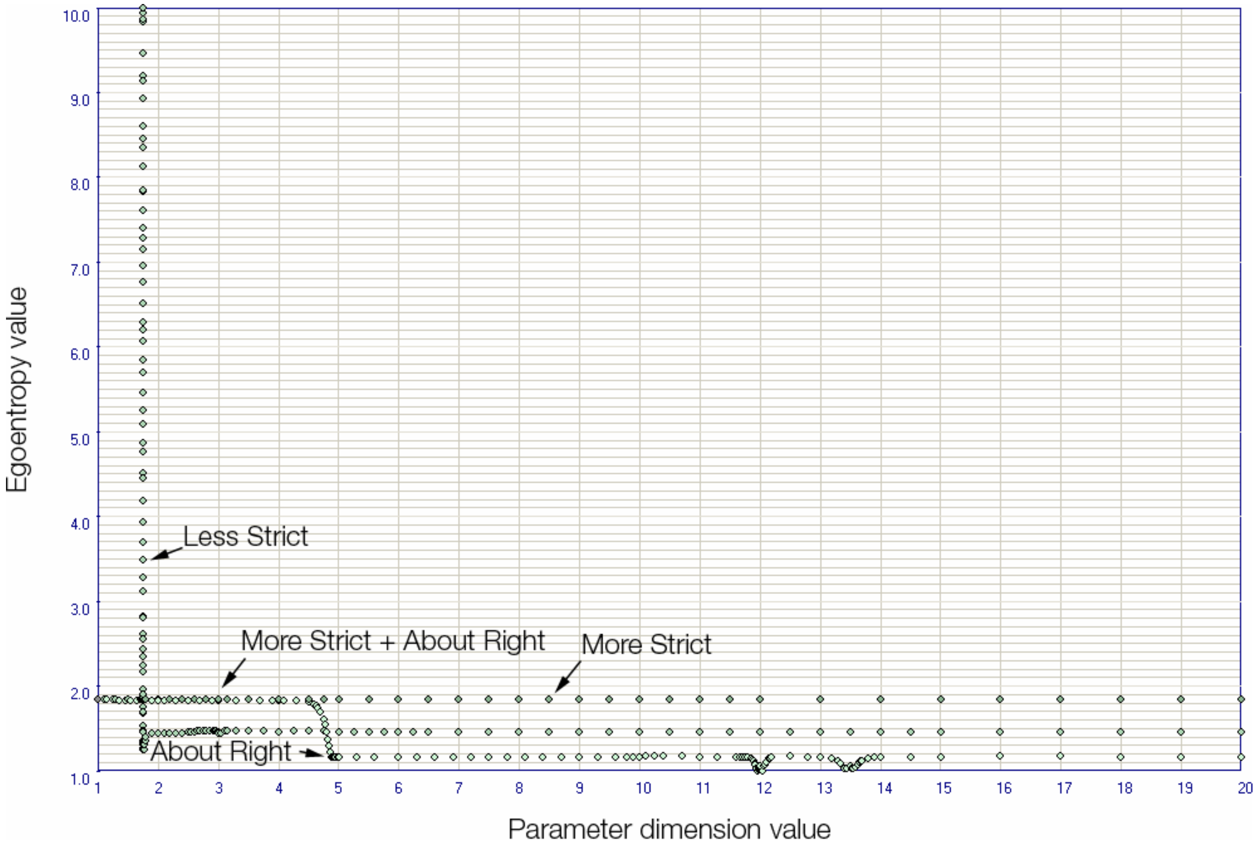

Figure 5. Dependence of egoentropy on a changing dimension value of one of the parameters, at two other parameters' constant dimension values of 1.

It has been experimentally established that when the starting dimension value of parameter "Less strict" was set at a constant level above 1.7, while the dimension values of parameters "More strict" and "About right" were set to monotonously increase, the system's egoentropy dynamics was dramatically different from the situation when the starting *DV* was the same for all the parameters: parameters "More strict" and "About right" were responsible for chaotic spikes similar to the one produced by parameter "Less strict" as shown in Fig. 5. All of the spikes had the same shape as shown in Fig. 2, i.e. each of them contained an antichaotic component. Positions of those spikes along the *DV* scale strongly depended on the initial *DV* of parameter "Less strict". The fact of such dependency makes it possible to establish whether or not there is a correlation between an initial dimension value of one of the parameters and positions of chaotic spikes emerging in response to variations in dimension values of the other two parameters.

Joint effect of parameters upon chaotic spike positions

In this experiment, we set the dimension of parameter "Less strict" to change within the *DV* range from 1.7 to 3.0 and registered positions of chaotic spikes emerging upon monotonous changes of dimension values of the other two parameters. The results are presented in Table 2.

Table 2. Positions of maximum points of chaotic spikes emerging upon variations of dimension values of parameters "More strict" and "About right" at certain initial dimensions values of parameter "Less strict".

LS[1)]	MS[2)]	AR[3)]
1.70	0	0
1.78	1.086	1.027
1.80	1.131	1.043
1.85	1.251	1.080
1.90	1.367	1.119
2.00	1.600	1.195
2.10	1.838	1.273
2.20	2.063	1.351
2.30	2.294	1.431
2.40	2.526	1.511
2.50	2.757	1.593
2.60	2.987	1.673
2.70	3.215	1.754
2.80	3.443	1.836
2.90	3.671	1.917
3.00	3.901	1.997

[1)] Initial DV of parameter "Less strict"

[2)] Maximum of chaotic spike emerging upon variations of DV of "More strict"

[3)] Maximum of chaotic spike emerging upon variations of DV of "About right"

As is shown in Fig. 6, positions of the peak maximums of the spikes resulting from the *DV* changes for parameters "More strict" and "About right", dependent on the initial *DV* of parameter "Less strict", are described by ideal linear curves whose intersection point corresponds to the initial *DV* of 1.744 for the "Less strict" parameter. As was mentioned earlier, this value exactly corresponds to the position of the chaotic spike emerging upon monotonous changes in the dimension value of parameter "Less strict", with parameters "More strict" and "About right" having constant initial *DV* of 1.

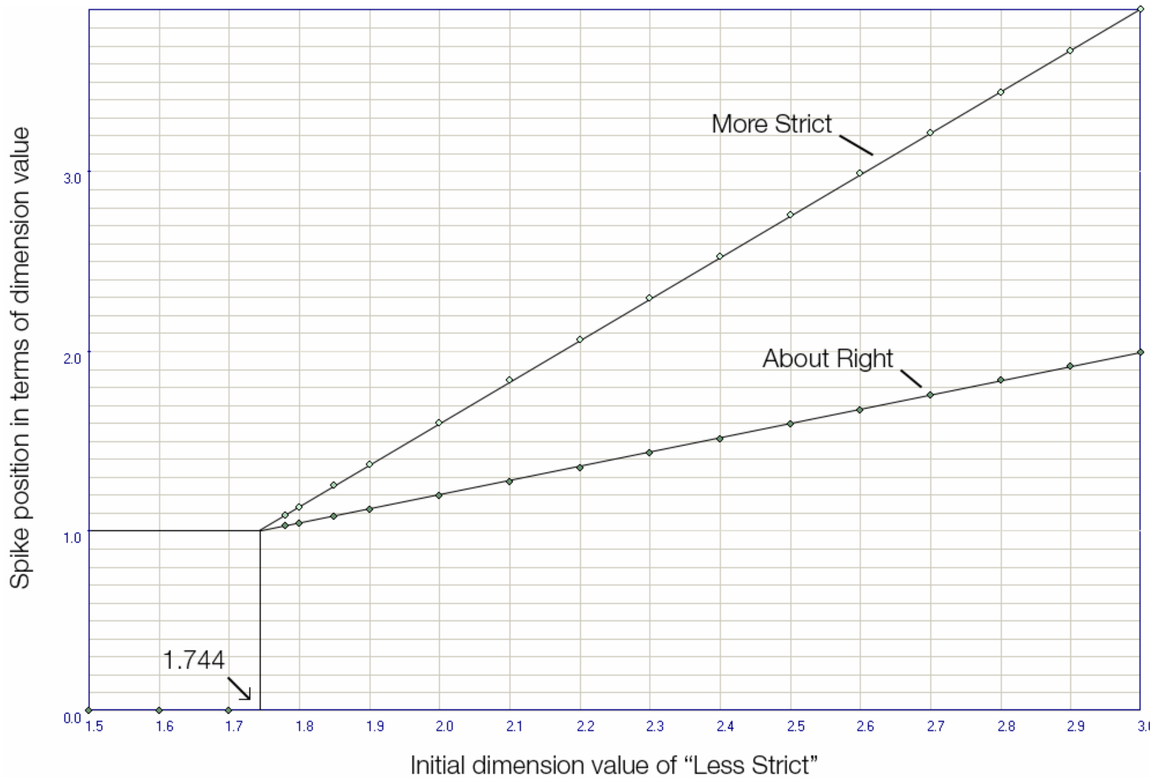

Figure 6. Relationship between initial dimension value of parameter "Less strict" and the positions of chaotic spikes caused by changes in dimension values of parameters "More strict" and "About right" (one of the parameters' *DV* monotonously changing while another's *DV* was constant and equal to 1).

These experimental findings are highly accurately described by the equations:

$$Max(MS) = 2.30LS - 3.00 \qquad (1),$$

and:

$$Max(AR) = 0.81LS - 0.41 \qquad (2),$$

Where *LS* is the initial dimension value of parameter "Less strict" that remains unchanged; and *Max(MS)* and *Max(AR)* are the maximum peaks of the chaotic spikes emerging upon monotonous changes in dimension values of parameters "More strict" and "About right".

Discussion

Fractal geometry of public opinion polls

The currently accepted approach to interpretation of public opinion polls is stunningly primitive: linear logic based on the arithmetical level of perception of data, coupled with classical mathematical statistical techniques to add solidity to analysis and as a means of unification of the professional communication code, thus making clear, for example, that at a certain confidence interval, 55% is more than 45%, whereas at a different confidence interval, 51% is hardly different from 49%. Contemporary methods for public opinion analysis are incapable of providing even short-term prediction of public opinion trends. Reliable predictions, if any, are not made by statistical analysis software, but only by human experts, based on subtleties that they perceive by the "sixth sense" developed after years of experience in analysis of public opinion polls.

The problem with sociological predictions based on public opinion polls is as follows. In reality, public opinion is a fractal space, rather than Euclidean into which the human mind is trying to put it because it does not know of any alternatives. To answer a question: "Will you vote for: (a) X; (b) Y; (c) none of the above", you have to express your opinion in integer dimensions – despite that your opinion of X and Y may be more complex: you may like X in principle but dislike some of his/her relevant features, and you may dislike Y in general but like some of his/her relevant features. As is known, the length of a coast can be measured in different scales, and, depending on a measurement scale, the result may considerably vary [6, 7]. Public opinion studies usually use an extremely crude scale of "yes – no" answers. It is only understandable that such "measurements" of public opinion are useless in prediction analysis. Real-life public opinion is a mosaic made of tiny details. It is, certainly, possible to design polls in a way that allows for fractalization of public opinion, providing the insight into respondents' genuine opinions – for instance, by including more specific questions; however, making predictions based on such answers would be even harder because there are not yet such methods to compare information obtained by fractalization of alternative questions of the "money or death" type.

As was earlier mentioned, this paper is not to propose better rules for interpretation of public opinion poll results or new methods for prediction of dynamics of public opinion – even though the results of this study provide solid grounds for that. We used national poll data as an example demonstrating that even very simple systems may present an analyst with very non-simple situations. The obtained results lead to two conclusions. Firstly, variations in dimensions of poll parameters set up extremely complex effects that are rooted in the fractal character of the phenomenon of public opinion. Changes in parameter dimensions can cause chaos in a system and make seemingly stable things suddenly change. For instance, the equilibrium between two opposite opinions, "More strict" and "Less strict", occurs when the dimension value of parameter "About right" reaches 0.43. The second conclusion is: changes in a system can be quantitatively determined and predicted, and the development of prediction methodology is technologically feasible. For instance, in the discussed case study, it is obvious that percentage of positive answers to the question "*do you think the gun control laws ought to be less strict?"* is three times more important for Democrats than it is for Republicans, because it has a strong influence on the generally dualistic position of Independents.

Chaos and antichaos

A typical chaotic spike and its anatomy are shown above in Figures 1 and 2, respectively. Based on analysis of a large number of various real-world and artificial systems, we have established that this is the most widely occurring type of chaotic spikes. Other forms are exceptionally rare, and they are represented by two types of chaotic situations, which will be discussed in another paper. One of them is connected with spontaneous self-oscillation processes emerging upon monotonous increase of parameters' dimensions. In such chaotic spikes, the ascending curve represents a sinusoid whose amplitude reaches the maximum at the mid-point of the ascent and is minimal at the point of the spike maximum. An inverse (antichaotic) spike, like the one demonstrated in Fig. 2, does not emerge in such cases. Also, the inverse spike may be absent in one of the two successive and overlapping chaotic spikes. In all other cases, the chaotic spike pattern is the same as in Fig. 2: at a certain level of egoentropy, close to maximum, an ascending peak collapses to a point close to the zero line of the egoentropy level, after which egoentropy starts increasing and, at a certain high point, drops again to a level close to zero.

Thus, to accurately describe a chaotic spike position, one can use three points of *DV*: the first inflection point (t_1); the point of the minimum egoentropy of the inverse spike (t_m); and the second inflection point (t_2). P_a value defines the point which corresponds to the inverse spike extremum normalized to the distance between the two inflection points:

$$P_a = \frac{t_m - t_1}{t_2 - t_1} \times 100 \qquad (3).$$

Typically, P_a values are within the range of 55 to 70%, although sometimes they may be as low as 30% or over 70%.

There are many arguments in favor of egoentropy [1] being a sensitive criterion that efficiently responds to changes in a system's complexity. The most convincing support for this claim is the following. Chaos spectra of complex systems, consisting of dozens of objects described by dozens of parameters, may change endlessly, depending on a parameter or combination of parameters undergoing the dimension changes, and may contain up to a dozen chaotic spikes with their maximums formed at different egoentropy levels. However, there is a rule that never changes in any system: a maximum egoentropy value (E_{max}) in a given system described by a given set of parameters is always the same. In paraphrase, whatever are the circumstances causing chaos in a system, the maximal chaos that can occur in that system is pre-determined and will never exceed a certain level defined by E_{max}. Our findings have shown that E_{max} of different systems may differ by twice in value.

As was mentioned above, the most credible explanation for the phenomenon of the inverse spike emerging in the area close to the culmination point of the chaotic spike seems to be the following: at a certain level of chaos in a system, further changes in a parameter dimension may result in emergence of order. In paraphrase, the inverse spike against a background of a chaotic spike is the manifestation of antichaos. An extremely important experimental finding established for all systems under chaos analysis is that the antichaotic component quickly fades out, and egoentropy dynamics returns to the tendency predetermined for a given system and expressed in its characteristic chaotic spike. Antichaos always yields to the predetermined chaos, apparently, due to its being dependent on certain local restructuring events that can only temporarily create order, whereas the chaotic peak describes the state of an entire system and the way it is expected to react at any point of monotonous changes in parameters' dimensions. Inevitability of transition from situation t_m to situation t_2 acquires a special emphasis in view of the fact that this regularity has been found to occur in most diverse systems, starting from various 3D geometrical structures, up to climatic, social, demographic, and many other systems. The concept of antichaos has been known since long ago and has been actively discussed in scientific literature for over 30 years by now. The antichaos concept is widely and irresponsibly used in speculations in biological and ecological sciences, aimed at "explanation" of the phenomenon of the living matter, as an effort to compensate for the lack of fundamental knowledge of how the life emerged and evolved. That being said, we should like to emphasize that the herein described form of manifestation of the antichaotic component of the process of chaos generalization has a universal character, which, along with examples based on experimental data, will be the subject of subsequent publications.

The above-described universal form of a chaotic spike emerging upon changes in the dimension of one of a system's parameters presents interest in the context of comparison with other experimentally investigated manifestations of chaos, described in other works on chaos study. It can also be an independent source of hypothesis in regard of many natural phenomena. For instance, in the context of the Big Bang theory – a gigantic cosmic explosion that is believed to have created the Universe – it seems plausible that the explosion occurred in the $t_1 - t_m$ section of the antichaotic spike (the Universe compression phase), and that currently the Universe is in the $t_m - t_2$ area (the Universe expansion phase). In the same way, it can be applied to the phenomenon of the irreversibility of time, and many other problems.

Appendix

Analysis of a system's chaos with MeaningFinder 2.2

The result demonstrated in Fig. 5 was obtained with *MeaningFinder 2.2*. Below is outlined the step-by-step procedure for analysis of chaos in any system of data points. Open the 'Tables' main window and download a table of data by importing or manual entry. To change a parameter *DV*, click on any cell of the respective column, then place the cursor on the 'Param' icon and click the mouse left button. In the 'Param' menu, select 'Set weight', this will open a dialog box 'Enter multiplication number' – type in a parameter weight and press OK. Click the 'Compute' icon, set a metric, and click the 'Go to similarity matrix' button – this will open the 'Similarity Matrix' window. Click the 'Egoentropy' icon to obtain cluster spectra of the system under analysis. The *E* value box on the upper bar will display the system's egoentropy value.

References

[1] Andreev, L. (2004) Concept of Egoentropy as a Measure of Complexity and Chaos. http://www.matrixreasoning.com/pdf/ConceptofEgoentropy.pdf

[2] Andreev, L. (2004) Cluster Spectroscopy. http://www.matrixreasoning.com/pdf/clusterspectroscopy.pdf

[3] Andreev, L. (2003) High-dimensional data clustering with the use of hybrid similarity matrices. U.S. Patent Application Ser. No. 10/622,542.

[4] Andreev, L. (2003) Unsupervised automated hierarchical data clustering based on simulation of a similarity matrix evolution. U.S. Patent No. 6,640,227.

[5] Andreev, L. (2004) *The Making of Non-Biological Intelligence. Part I. Supplement to MeaningFinder™*. http://www.matrixreasoning.com/pdf/chapter1.pdf

[6] Richardson, L. (1961) The Problem of contiguity. An appendix of statistics of deadly quarrels. General Systems Yearbook 6, pp. 139-187.

[7] Mandelbrot, B. B. (1977) *The Fractal Geometry of Nature*. W. H. Freeman and Co., NY